Independence of R.V.

Recall that x and y are independent if $P(xy) = P(x) \cdot P(y)$.

1. Definition:

Let x and y be two r.v. X and Y are independent if $P(x \in B, y \in B) = P(x \in B) \cdot P(y \in B)$. We write $x \perp y$ to show that x and y are independent.

Note: $P(a \le x \le b, c \le y \le d) = P(a \le x \le b) \cdot P(c \le y \le d)$ whenever $a \le b$, $c \le d$.

Note: If x and y are jointly discrete, then $x \perp y$ iff $P_{x,y}(x,y) = P_x(x) \cdot P_y(y)$ $\forall x, y \in R$.

Note: If x and y are jointly abs cont, then $x \perp y$ iff $f_{x,y}(x,y) = f_x(x) \cdot f_y(y)$, $\forall x, y \in R$.

E.g.

$$f_{x,y}(x,y) = \begin{cases} 4xy, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Are x and y independent?

$$f_x(x) = 2x$$
$$f_y(y) = 2y$$

$$f_x(x) \cdot f_y(y) = (2x)(2y) = 4xy = f_{x,y}(x,y)$$

$\therefore$ $x \perp y$, $\forall x, y \in R$

Recall that x and y are independent if $P(x|y) = P(x)$.

2. Independence of Conditional Probabilities:

Let x and y be two r.v. Then,:

1. If x and y are jointly discrete, then $x \perp y$ iff $P_{x|y}(x|y) = P_x(x)$ $\forall x, y \in \mathbb{R}$ s.t. $P_y(y) > 0$.

2. If x and y are jointly abs cont, then $x \perp y$ iff $f_{x|y}(x|y) = f_x(x)$ $\forall x, y \in \mathbb{R}$ s.t. $f_y(y) > 0$.