B52 Distributions

1. Degenerate Distribution:
   A random variable, x, is degenerate if for some c, $P(x=c)=1$ and $P(x \neq c)=0$.

   E.g. Suppose there exists a 6-sided die s.t. all the faces show 6. Then, $P(x=6)=1$ and $P(x \neq 6)=0$.

2. Binominal Distribution:
   A binominal distribution only has 2 possible outcomes, success and failure. A success is when you get the result you want and a failure is when you don't get the result you want.

   Furthermore, binominal distributions must satisfy these conditions:
   1. The number of trials is fixed.
   2. Each trial is independent.
   3. The probability of success remains the same for all trials

   Binominal distributions are denoted by $X \sim \text{Binominal}(n, \theta)$

   The formula is $P_x(x) = \binom{n}{x} \theta^x (1-\theta)^{n-x}$ where
   1. n is the number of trials
   2. θ is the prob of success
   3. x is the number of successes

E.g. A fair coin is tossed 10 times. What is the probability of getting exactly 6 heads?

In this case,

$n = 10$

$\theta = \frac{1}{2}$

$x = 6$

$$P_x(6) = \binom{10}{6}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4$$

$$= 0.21 \text{ or } 21\%$$

3. Bernouli Distribution:

A bernouli distribution has 2 outcomes, success and failure. Furthermore, its sole condition is that each trial is independent.

Bernouli distributions are denoted by $x \sim \text{Bernouli}(\theta)$

The formula is $P_x(x) = \begin{cases} \theta, & \text{if success} \\ 1-\theta, & \text{if failure} \end{cases}$

where θ is the probability of success.

E.g. If you flip a coin, what's the probability of getting Head?

$\theta = \frac{1}{2}$

$$P_x(\text{Head}) = \begin{cases} \frac{1}{2}, & \text{Head} \\ \frac{1}{2}, & \text{Tail} \end{cases}$$

4. Geometric Distribution:

This represents the number of failures before getting your first success.

It must satisfy these 3 conditions:

1. There are only 2 outcomes, success and failure.
2. The trials are independent.
3. The probability of success is the same for all trials.

It is denoted by $X \sim Geo(\theta)$

The formula is $P_X(x) = (1-\theta)^x (\theta)$ where

1. θ is the probability of success.
2. X is the number of failures before the first success.

E.g. If you flip a coin 3 times, what is the probability that you will get your first head on your 3rd flip?

$\theta = \frac{1}{2}$

$X = 2$ (Bc you're getting your first head on your 3rd try.)

$P(X = \text{First head on 3rd flip}) = (1-\frac{1}{2})^2 (\frac{1}{2})$

$= \frac{1}{8}$

5. Negative Binominal Distribution:
This represents the number of failures before getting your r^{th} success. This is a generalized version of the Geometric Distribution.

Furthermore, it must satisfy these conditions:
1. Each trial is independent.
2. There are only 2 outcomes, success and failure.
3. The probability of success is the same for all trials.

This is denoted by $X \sim$ Neg-Binominal(r, θ)

The formula is $P_X(x) = ({}_{r-1+k}C_k)(\theta)^r(1-\theta)^k$ where
1. r is the number of successes.
2. k is the number of failures.
3. θ is the probability of success.

E.g. If you flip a coin 3 times, what is the probability that you will get Heads on your 3rd try?

$\theta = \frac{1}{2}$

$r = 1$

$k = 2$

$$P_X(x) = ({}_2C_2)(\tfrac{1}{2})^1(\tfrac{1}{2})^2$$
$$= \tfrac{1}{8}$$

As shown, Neg-Binominal = Geo if $r=1$.

6. Hypergeometric Distribution:

This represents the probability of drawing n items from N objects without replacing them.

This is denoted by $X \sim Hyper(N, m, n)$

The formula is $P_X(x) = \dfrac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$ where

1. M is the number of successes in the population.
2. X is the number of observed successes.
3. N is the population size.
4. n is the number of draws.

E.g. A deck contains 20 cards, 6 red and 14 black. 5 cards are drawn without replacement. What is the probability that exactly 4 red cards are drawn?

$M = 6$

$X = 4$

$N = 20$

$n = 5$

$$P_X(x) = \frac{\binom{6}{4}\binom{20-6}{5-4}}{\binom{20}{5}}$$

$$= 0.0135$$

7. Poisson Distribution:

This predicts the probability of certain events happening given the fact of how often this event occurred.

It is denoted by $X \sim \text{Poisson}(\lambda)$

The formula is $P_X(x) = \frac{(e^{-\lambda})(\lambda^x)}{x!}$ where

1. λ is the expected number of occurences.
2. x is the predicated number of future occurences.

E.g. A city experiences 2 major storms per year on average. What is the probability that there will be exactly 3 major storms in that city this year?

$\lambda = 2$

$x = 3$

$$P_X(x) = \frac{(e^{-2})(2^3)}{3!}$$

$$= 0.180$$