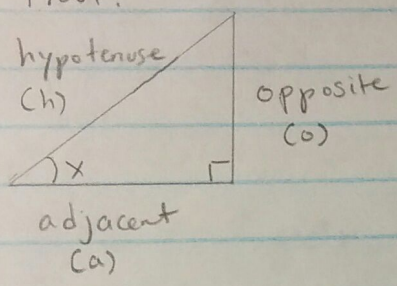
Trig Proofs and Laws

1. $\cos^2 x + \sin^2 x = 1$

Proof:



$\sin x = \frac{o}{h}$ $\cos x = \frac{a}{h}$

$LS = \sin^2 x + \cos^2 x$

$= \left(\frac{o}{h}\right)^2 + \left(\frac{a}{h}\right)^2$

$= \frac{o^2 + a^2}{h^2}$

$= \frac{h^2}{h^2}$

$= 1$

$LS = RS$

QED

2. $\tan^2 x + 1 = \sec^2 x$

Proof:

$\sin^2 x + \cos^2 x = 1$

If we divide both sides by $\cos^2 x$, we get

$\tan^2 x + 1 = \sec^2 x$

QED

3 $\cot^2 x + 1 = \csc^2 x$

Proof:

$\sin^2 x + \cos^2 x = 1$

If we divide both sides by $\sin^2 x$, we get

$\cot^2 x + 1 = \csc^2 x$

QED

4. $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

Proof:

1. $\sin(x) = \cos(x - \frac{\pi}{2})$
2. $-\cos(x) = \sin(x - \frac{\pi}{2})$
3. $-\sin(x) = \cos(x + \frac{\pi}{2})$
4. $\cos(x) = \sin(x + \frac{\pi}{2})$

$\sin(x+y) = \cos(x+y-\frac{\pi}{2})$

$= \cos(x + (y - \frac{\pi}{2}))$

$= \cos x \cos(y - \frac{\pi}{2}) - \sin x \sin(y - \frac{\pi}{2})$

$= \cos x \sin y - \sin x(-\cos y)$

$= \sin x \cos y + \cos x \sin y$

QED

$\sin(x-y) = \cos(x-y-\frac{\pi}{2})$

$= \cos(x - (y + \frac{\pi}{2}))$

$= \cos x \cos(y + \frac{\pi}{2}) + \sin x \sin(y + \frac{\pi}{2})$

$= \cos x(-\sin y) + \sin x \cos y$

$= \sin x \cos y - \cos x \sin y$

QED

5. $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\cos(x+y) = \sin(x+y+\frac{\pi}{2})$

$= \sin(x+(y+\frac{\pi}{2}))$

$= \sin x \cos(y+\frac{\pi}{2}) + \sin(y+\frac{\pi}{2})\cos x$

$= \sin x(-\sin y) + \cos x \cos y$

$= \cos x \cos y - \sin x \sin y$

QED

$\cos(x-y) = \sin(x-y+\frac{\pi}{2})$

$= \sin(x-(y-\frac{\pi}{2}))$

$= \sin x \cos(y-\frac{\pi}{2}) - \cos x \sin(y-\frac{\pi}{2})$

$= \sin x(\sin y) - \cos x(-\cos y)$

$= \cos x \cos y + \sin x \sin y$

QED

6. $\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

Proof:

$\tan(x+y) = \dfrac{\sin(x+y)}{\cos(x+y)}$

$= \dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}$ Divide by $\cos x \cos y$ Divide by $\cos x \cos y$

$= \dfrac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$

QED

$\tan(x-y) = \dfrac{\sin(x-y)}{\cos(x-y)}$

$= \dfrac{\sin x \cos y - \sin y \cos x}{\cos x \cos y + \sin x \sin y}$ (Divide by $\cos x \cos y$) (Divide by $\cos x \cos y$)

$= \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

QED

7.

$\sin(2x) = 2\sin x \cos x$

Proof:

$\sin(2x) = \sin(x+x)$

$= \sin x \cos x + \sin x \cos x$

$= 2\sin x \cos x$

QED

8. $\cos(2x) = \cos^2 x - \sin^2 x$

$= 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

Proof:

$\cos(2x) = \cos(x+x)$

$= \cos x \cos x - \sin x \sin x$

$= \cos^2 x - \sin^2 x$

$= \cos^2 x - (1 - \cos^2 x)$ OR $= (1 - \sin^2 x) - \sin^2 x$

$= 2\cos^2 x - 1$ $= 1 - 2\sin^2 x$

QED

9. $\tan(2x) = \dfrac{2\tan x}{1 - \tan^2 x}$

Proof:

$\tan(2x) = \dfrac{\sin(2x)}{\cos(2x)}$

$= \dfrac{2\sin x \cos x}{\cos^2 x - \sin^2 x}$ (Divide by $\cos^2 x$) (Divide by $\cos^2 x$)

$= \dfrac{2\tan x}{1 - \tan^2 x}$

QED

10. $\sin^2\left(\frac{x}{2}\right) = \frac{1-\cos x}{2}$

Proof:

We know that $\sin^2\left(\frac{x}{2}\right) + \cos^2\left(\frac{x}{2}\right) = 1$

$\sin^2\left(\frac{x}{2}\right) = 1 - \cos^2\left(\frac{x}{2}\right)$

$= 1 - \left(\frac{1+\cos x}{2}\right)$

$= \frac{2-1-\cos x}{2}$

$= \frac{1-\cos x}{2}$

QED

11. $\cos^2\left(\frac{x}{2}\right) = \frac{1+\cos x}{2}$

Proof:

$\sin^2\left(\frac{x}{2}\right) + \cos^2\left(\frac{x}{2}\right) = 1$

$\cos^2\left(\frac{x}{2}\right) = 1 - \sin^2\left(\frac{x}{2}\right)$

$= 1 - \left(\frac{1-\cos x}{2}\right)$

$= \frac{2-1+\cos x}{2}$

$= \frac{1+\cos x}{2}$

QED

Summary of Trig Laws

1. $\cos^2 x + \sin^2 x = 1$
2. $\tan^2 x + 1 = \sec^2 x$
3. $\cot^2 x + 1 = \csc^2 x$
4. $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
5. $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
6. $\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$
7. $\sin(2x) = 2 \sin x \cos x$
8. $\cos(2x) = \cos^2 x - \sin^2 x$
   $= 2\cos^2 x - 1$
   $= 1 - 2\sin^2 x$
9. $\tan(2x) = \dfrac{2\tan x}{1 - \tan^2 x}$
10. $\sin^2\left(\frac{x}{2}\right) = \dfrac{1 - \cos x}{2}$
11. $\cos^2\left(\frac{x}{2}\right) = \dfrac{1 + \cos x}{2}$
12. $\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$
13. $-\cos(x) = \sin\left(x - \frac{\pi}{2}\right)$
14. $-\sin(x) = \cos\left(x + \frac{\pi}{2}\right)$
15. $\cos(x) = \sin\left(x + \frac{\pi}{2}\right)$
16. $\sin(-x) = -\sin(x)$ (odd Function)
17. $\cos(-x) = \cos x$ (Even Function)
18. $\tan(-x) = -\tan x$ (odd Function)

Principal Values

1. $\sin x$: $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
2. $\cos x$: $x \in [0, \pi]$
3. $\tan x$: $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$
4. $\csc x$: $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $x \neq \pi$
5. $\sec x$: $x \in [0, \pi]$, $x \neq \frac{\pi}{2}$
6. $\cot x$: $x \in (0, \pi)$