Math Proofs and Laws

Log Proofs

1. Product Rule

$\log_b(x \cdot y) = \log_b x + \log_b y$

Proof:

Let x be b^x.

Let y be b^y.

$LS = \log_b(b^x \cdot b^y)$

$= \log_b(b^{x+y})$

$= x+y$

$RS = \log_b(b^x) + \log_b(b^y)$

$= x+y$

$LS = RS$

QED

2. Quotient Rule

$\log_b(\frac{x}{y}) = \log_b x - \log_b y$

Proof:

Let x be b^x

Let y be b^y

$LS = \log_b(\frac{b^x}{b^y})$

$= \log_b(b^{(x-y)})$

$= x-y$

$RS = \log_b(b^x) - \log_b(b^y)$

$= x-y$

$LS = RS$

QED

3. Power Rule

$\log_b x^y = y(\log_b x)$

Proof:

Let x be b^x

LS $= \log_b (b^{xy})$

$= \log_b (b^{yx})$

$= yx$

RS $= y(\log_b (b^x))$

$= yx$

LS = RS

QED

4. Change of Base Rule

$\log_b x = \frac{\log_c x}{\log_c b}$

Proof:

Let $x = b^x$

LS $= \log_b (b^x)$

$= x$

RS $= \frac{\log_c (b^x)}{\log_c b}$

$= \frac{x(\log_c b)}{\log_c b}$

$= x$

LS = RS

QED

Summary of Log Laws

1. Product Law
$\log_b(xy) = \log_b x + \log_b y$

2. Quotient Law
$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

3. Power Law
$\log_b(x^y) = y(\log_b x)$

4. Change of Base Law
$\log_b x = \frac{\log_c x}{\log_c b}$ A special case of this rule: $\log_b x = \frac{\log x}{\log x^b} = \frac{1}{\log x^b}$

5. Log Base Law
$\log_b b = 1$

6. Log of 1 Law
$\log_b 1 = 0$

7. Restrictions
In $\log_b x$, x must be greater than 0. $\log_b x$ will be undefined if $x \leq 0$.
Furthermore, if $0 < x < 1$, $\log_b x$ will be negative.