Limit Laws

1. $\lim_{x \to c} x = c$

Proof:

$\forall \varepsilon > 0, \exists d > 0 \mid 0 < |x-c| < d \rightarrow |x-c| < \varepsilon$

Choose d to be $x-c$

$|x-c| < \varepsilon$

$d = \varepsilon$

QED

2. $\lim_{x \to c} k = k$, k is a constant

Proof:

$\forall \varepsilon > 0, \exists d > 0 \mid 0 < |x-c| < d \rightarrow |k-k| < \varepsilon$

3. $\lim_{x \to c} mx+b = mc+b$

$\forall \varepsilon > 0, \exists d > 0 \mid 0 < |x-c| < d \rightarrow |mx+b-(mc+b)| < \varepsilon$

$|mx+b-(mc+b)|$

$= |mx+b-mc-b|$

$= |mx-mc|$

$= |m(x-c)|$

$= m|x-c|$

$= md$

$md = \varepsilon$

$d = \frac{\varepsilon}{m}$

$|mx+b-(mc+b)| < \varepsilon$

$|m(x-c)| < \varepsilon$

$m\left(\frac{\varepsilon}{m}\right) = \varepsilon$

QED

4. Sum and Difference Law

$$\lim_{x \to a} [f \pm g] = \lim_{x \to a} f \pm \lim_{x \to a} g$$

5. Constant Multiplier Law

$$\lim_{x \to a} cf = c \lim_{x \to a} f$$

6. Product Law

$$\lim_{x \to a} [fg] = \left(\lim_{x \to a} f \right)\left(\lim_{x \to a} g \right)$$

7. Quotient Law

$$\lim_{x \to a} \left[\frac{f}{g} \right] = \frac{\lim_{x \to a} f}{\lim_{x \to a} g}, \quad \lim_{x \to a} g \neq 0$$

8. Power Law

$$\lim_{x \to a} (f)^n = \left(\lim_{x \to a} f \right)^n$$

9. Squeeze Theorem

If $h(x) \leq f(x) \leq g(x)$ and $\lim_{x \to a} h(x) = L$ and $\lim_{x \to a} g(x) = L$, then

$$\lim_{x \to a} f(x) = L$$